Debt Sustainability Condition ±Mathematical Version

The famous debt sustainability condition is often written mathematically as follows:

¿ l n š / l š · š p L :˜ š F • š; l n š? Ú / l š? Ú · š? Ú p E s š F € š / l š · š F ¿t š / l š · š

SIMPLE ANALYTICS:

Public Debt-to-GDP ratio will rise if:

˜ š P • š

s š F € š P Ù

Debt sustainability requires:

¿ l n š / l š · š p L Ù

If debt monetization is pursued ¿* ç P r, it will help reduce the debt burden. However, debt monetization may result in higher inflation. If we rule out debt monetization (¿* ç L r), then debt sustainability requires:

F l s š F € š / l š · š p L :˜ š F • š; l n š? Ú / l š? Ú · š? Ú p

or, !0ˆ^ y.ìÍ å y.¿1/ „˜CW •, à Kä a''ëó ñ I„5qp R0 ÄÓ a''ëó ñ I„5qp p† à >@à y.¿1/ •" à

l € š F s š / l š · š p L :˜ š F • š; l n š? Ú / l š? Ú · š?

) ç: Government Purchases

6 ç: Taxes net of Transfers (6 ç L 6 = T A F 6 N = J O) A N O

) ç F 6 ç: Primary Budget Deficit (note:) ç P 6 ç implies a primary budget deficit)

$ ç ã Government Debt in Period t

DERIVATION

Reference: Macroeconomics (2nd Edition) by R. Glenn Hubbard and Anthony Patrick O'Brien (Publisher: Pearson)

Standard budget deficit is given by:

$$D_t = i_t B_{t-1} + G_t - T_t$$

To capture the role of seignorage (essentially represents a transfer of wealth from individuals holding money to the government), we extend the above equation to include change in monetary base:

$$D_t = i_t B_{t-1} + G_t - T_t - \Delta M_t$$

Note that the change in government debt is given by:

$$\Delta B_t = B_t - B_{t-1} = D_t = i_t B_{t-1} + G_t - T_t - \Delta M_t$$

Divide both sides by nominal GDP and rearrange to get

$$\frac{B_t}{P_t Y_t} = (1 + i_t)\frac{B_{t-1}}{P_t Y_t} + \frac{G_t}{P_t Y_t} - \frac{T_t}{P_t Y_t} - \frac{\Delta M_t}{P_t Y_t}$$

Modify pervious equation as follows (multiply and divide the first term on the right hand side by $P_{t-1} Y_{t-1}$):

$$\frac{B_t}{P_t Y_t} = (1 + i_t)\left(\frac{P_{t-1} Y_{t-1}}{P_t Y_t}\right)\left(\frac{B_{t-1}}{P_{t-1} Y_{t-1}}\right) + \frac{G_t}{P_t Y_t} - \frac{T_t}{P_t Y_t} - \frac{\Delta M_t}{P_t Y_t}$$

Note the following condition,

$$\left(\frac{P_t Y_t}{P_{t-1} Y_{t-1}}\right) = \frac{P_t}{P_{t-1}} \frac{Y_t}{Y_{t-1}} = (1 + g_t)(1 + i_t)$$

$$\left(\frac{P_t Y_t}{P_{t-1} Y_{t-1}}\right) = (1 + g_t)(1 + i_t) \approx 1 + g_t + i_t$$

Using the approximation

$$\left(\frac{P_t Y_t}{P_{t-1} Y_{t-1}}\right) \approx 1 + g_t + i_t$$

We get:

$$\frac{B_t}{P_t Y_t} = \left(\frac{1 + i_t}{1 + g_t + i_t}\right)\left(\frac{B_{t-1}}{P_{t-1} Y_{t-1}}\right) + \frac{G_t}{P_t Y_t} - \frac{T_t}{P_t Y_t} - \frac{\Delta M_t}{P_t Y_t}$$

Another useful approximation:

l ÚE •š / ÚE Êš E • š p N ÚE •š F Êš F • š

Sα

nš / | š··š L :ÚE •š F Êš F • š; l nš?Ú / | š?Ú··š?Ú p E sš / | š··š F €š / | š··š F ¿t š / | š··š

Rearrange terms to get: